FOCUS ACADEMY

Kg to 12

English&Gujarati Medium

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Class 9 Maths Chapter 2 Polynomials

Ex 2.1

Ex 2.1 Class 9 Maths Question 1.
Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.
(i) $4x^2 – 3x + 7$
(ii) $y^2 + \sqrt{2}$
(iii) $3 \sqrt{t} + t\sqrt{2}$
(iv) y+ 2y
(v) $x^{10}+ y^3+t^{50}$
Solution:
(i) We have $4x^2 – 3x + 7 = 4x^2 – 3x + 7x^0$
It is a polynomial in one variable i.e., x
because each exponent of x is a whole number.
(ii) We have $y^2 + \sqrt{2} = y^2 + \sqrt{2}y^0$
It is a polynomial in one variable i.e., y
because each exponent of y is a whole number.

(iii) We have $3 \sqrt{t} + t\sqrt{2} = 3 \sqrt{t}^{1/2} + \sqrt{2}.t$
It is not a polynomial, because one of the exponents of t is 12,
which is not a whole number.
(iv) We have y + y+2y = $y + 2.y^{-1}$
It is not a polynomial, because one of the exponents of y is -1,
which is not a whole number.
(v) We have $x^{10}+ y^3 + t^{50}$
Here, exponent of every variable is a whole number, but $x^{10} + y^3 + t^{50}$ is a polynomial in x, y and t,

i.e., in three variables.
So, it is not a polynomial in one variable.

Ex 2.1 Class 9 Maths Question 2.
Write the coefficients of x^2 in each of the following
(i) $2 + x^2 + x$
(ii) $2 – x^2 + x^3$
(iii) $\pi 2\ x^2 + x$
(iv) $\sqrt{2}\ x – 1$
Solution:
(i) The given polynomial is $2 + x^2 + x$.
The coefficient of x^2 is 1.
(ii) The given polynomial is $2 – x^2 + x^3$.
The coefficient of x^2 is -1.
(iii) The given polynomial is $\pi 2 X2 + x$.
The coefficient of x^2 is $\pi 2$.
(iv) The given polynomial is $\sqrt{2}\ x – 1$.
The coefficient of x^2 is 0.
Ex 2.1 Class 9 Maths Question 3.
Give one example each of a binomial of degree 35, and of a monomial of degree 100.
Solution:
(i) Abmomial of degree 35 can be $3x^{35}$ -4.
(ii) A monomial of degree 100 can be $\sqrt{2}y^{100}$.

Ex 2.1 Class 9 Maths Question 4.
Write the degree of each of the following polynomials.
(i) $5x^3+4x^2 + 7x$
(ii) $4 – y^2$
(iii) $5t – \sqrt{7}$
(iv) 3
Solution:
(i) The given polynomial is $5x^3 + 4x^2 + 7x$.
The highest power of the variable x is 3.
So, the degree of the polynomial is 3.
(ii) The given polynomial is $4- y^2$. The highest
power of the variable y is 2.
So, the degree of the polynomial is 2.
(iii) The given polynomial is $5t – \sqrt{7}$. The highest power of variable t is 1. So, the degree of the polynomial is 1.
(iv) Since, $3 = 3x^\circ$ [$\because x^\circ=1$]
So, the degree of the polynomial is 0.

Ex 2.1 Class 9 Maths Question 5.
Classify the following as linear, quadratic and cubic polynomials.
(i) $x^2+ x$
(ii) $x – x^3$

(iii) $y + y^2+4$
(iv) $1 + x$
(v) $3t$
(vi) r^2
(vii) $7x^3$
Solution:
(i) The degree of $x^2 + x$ is 2. So, it is a quadratic polynomial.
(ii) The degree of $x – x^3$ is 3. So, it is a cubic polynomial.
(iii) The degree of $y + y^2 + 4$ is 2. So, it is a quadratic polynomial.
(iv) The degree of $1 + x$ is 1. So, it is a linear polynomial.
(v) The degree of $3t$ is 1. So, it is a linear polynomial.
(vi) The degree of r^2 is 2. So, it is a quadratic polynomial.
(vii) The degree of $7x^3$ is 3. So, it is a cubic polynomial.

NCERT Solutions for Class 9 Maths Chapter 2 Polynomials Ex 2.2

Question 1.
Find the value of the polynomial $5x – 4x^2 + 3$ at
(i) $x = 0$
(ii) $x = – 1$
(iii) $x = 2$
Solution:
1et $p(x) = 5x – 4x^2 + 3$
(i) $p(0) = 5(0) – 4(0)^2 + 3 = 0 – 0 + 3 = 3$
Thus, the value of $5x – 4x^2 + 3$ at $x = 0$ is 3.
(ii) $p(-1) = 5(-1) – 4(-1)^2 + 3$
$= – 5x – 4x^2 + 3 = -9 + 3 = -6$
Thus, the value of $5x – 4x^2 + 3$ at $x = -1$ is -6.
(iii) $p(2) = 5(2) – 4(2)^2 + 3 = 10 – 4(4) + 3$
$= 10 – 16 + 3 = -3$
Thus, the value of $5x – 4x^2 + 3$ at $x = 2$ is – 3.

Question 2.
Find p (0), p (1) and p (2) for each of the following polynomials.
(i) $p(y) = y^2 – y +1$
(ii) $p (t) = 2 +1 + 2t^2 -t^3$
(iii) $P (x) = x^3$
(iv) $p (x) = (x-1) (x+1)$
Solution:
(i) Given that $p(y) = y^2 – y + 1$.
$\therefore P(0) = (0)^2 – 0 + 1 = 0 – 0 + 1 = 1$
$p(1) = (1)^2 – 1 + 1 = 1 – 1 + 1 = 1$
$p(2) = (2)^2 – 2 + 1 = 4 – 2 + 1 = 3$
(ii) Given that $p(t) = 2 + t + 2t^2 – t^3$

∴$p(0) = 2 + 0 + 2(0)^2 – (0)^3$
$= 2 + 0 + 0 – 0=2$
$P(1) = 2 + 1 + 2(1)^2 – (1)^3$
$= 2 + 1 + 2 – 1 = 4$
$p(2) = 2 + 2 + 2(2)^2 – (2)^3$
$= 2 + 2 + 8 – 8 = 4$
(iii) Given that $p(x) = x^3$
∴ $p(0) = (0)^3 = 0$, $p(1) = (1)^3 = 1$
$p(2) = (2)^3 = 8$
(iv) Given that p(x) = (x – 1)(x + 1)
∴ p(0) = (0 – 1)(0 + 1) = (-1)(1) = -1
p(1) = (1 – 1)(1 +1) = (0)(2) = 0
P(2) = (2 – 1)(2 + 1) = (1)(3) = 3

Question 3.
Verify whether the following are zeroes of the polynomial, indicated against them.
(i) p(x) = 3x + 1,x = –13
(ii) p (x) = 5x – π, x = 45
(iii) p (x) = x^2 – 1, x = x – 1
(iv) p (x) = (x + 1) (x – 2), x = – 1,2
(v) p (x) = x^2, x = 0
(vi) p (x) = 1x + m, x = – m1
(vii) P (x) = $3x^2$ – 1, x = – 13√,23√
(viii) p (x) = 2x + 1, x = 12
Solution:
(i) We have , p(x) = 3x + 1

$$\therefore \quad p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0$$

Since, $p\left(-\frac{1}{3}\right) = 0$, so, $x = -\frac{1}{3}$ is a zero of $3x + 1$.

(ii) We have, p(x) = 5x – π
∴ p(−13)=3(−13)+1=−1+1=0
(iii) We have, $p(x) = x^2 – 1$
∴ $p(1) = (1)^2 – 1 = 1 – 1=0$
Since, p(1) = 0, so x = 1 is a zero of x^2 -1.
Also, $p(-1) = (-1)^2 -1 = 1 – 1 = 0$
Since p(-1) = 0, so, x = -1, is also a zero of $x^2 – 1$.
(iv) We have, p(x) = (x + 1)(x – 2)
∴ p(-1) = (-1 +1) (-1 – 2) = (0)(- 3) = 0
Since, p(-1) = 0, so, x = -1 is a zero of (x + 1)(x – 2).
Also, p(2) = (2 + 1)(2 – 2) = (3)(0) = 0
Since, p(2) = 0, so, x = 2 is also a zero of (x + 1)(x – 2).

(v) We have, $p(x) = x^2$

$\therefore p(o) = (0)^2 = 0$

Since, p(0) = 0, so, x = 0 is a zero of x^2.

(vi) We have, p(x) = lx + m

$$\therefore \quad p\left(-\frac{m}{l}\right) = l\left(-\frac{m}{l}\right) + m = -m + m = 0.$$

Since, $p\left(-\frac{m}{l}\right) = 0$, so, $x = -\frac{m}{l}$ is a zero of $lx + m$.

(vii) We have, $p(x) = 3x^2 – 1$

$$\therefore \quad p\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{1}{3}\right) - 1 = 1 - 1 = 0$$

Since, $p\left(-\frac{1}{\sqrt{3}}\right) = 0$, so, $x = -\frac{1}{\sqrt{3}}$ is a zero of $3x^2 - 1$.

Also, $p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{4}{3}\right) - 1$

$= 4 - 1 = 3$

Since, $p\left(\frac{2}{\sqrt{3}}\right) \neq 0$, so, $\frac{2}{\sqrt{3}}$ is not a zero of $3x^2 - 1$.

(viii) We have, p(x) = 2x + 1

∴ p(12)=2(12)+1=1+1=2

Since, p(12) ≠ 0, so, x = 12 is not a zero of 2x + 1.

Question 4.

Find the zero of the polynomial in each of the following cases

(i) p(x)=x+5

(ii) p (x) = x – 5

(iii) p (x) = 2x + 5

(iv) p (x) = 3x – 2

(v) p (x) = 3x

(vi) p (x)= ax, a≠0

(vii) p (x) = cx + d, c ≠ 0 where c and d are real numbers.

Solution:

(i) We have, p(x) = x + 5. Since, p(x) = 0

⇒ x + 5 = 0

⇒ x = -5.
Thus, zero of x + 5 is -5.

(ii) We have, p(x) = x – 5.
Since, p(x) = 0 ⇒ x – 5 = 0 ⇒ x = -5
Thus, zero of x – 5 is 5.

(iii) We have, p(x) = 2x + 5. Since, p(x) = 0
⇒ 2x + 5 =0
⇒ 2x = -5
⇒ x = −52
Thus, zero of 2x + 5 is −52 .
(iv) We have, p(x) = 3x – 2. Since, p(x) = 0
⇒ 3x – 2 = 0
⇒ 3x = 2
⇒ x = 23
Thus, zero of 3x – 2 is 23
(v) We have, p(x) = 3x. Since, p(x) = 0
⇒ 3x = 0 ⇒ x = 0
Thus, zero of 3x is 0.

(vi) We have, p(x) = ax, a ≠ 0.
Since, p(x) = 0 => ax = 0 => x-0
Thus, zero of ax is 0.

(vii) We have, p(x) = cx + d. Since, p(x) = 0
⇒ cx + d = 0 ⇒ cx = -d ⇒ x=−dc
Thus, zero of cx + d is −dc

Ex 2.3

Question 1.
Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by
(i) x + 1
(ii) x – 12
(iii) x
(iv) x + π
(v) 5 + 2x
Solution:
Let $p(x) = x^3 + 3x^2 + 3x +1$
(i) The zero of x + 1 is -1.
∴ p(-1) = (-1)3 + 3(-1)2 + 3(-1) +1
= -1 + 3- 3 + 1 = 0
Thus, the required remainder = 0

(ii) The zero of $x-\frac{1}{2}$ is $\frac{1}{2}$

$$\therefore \quad p\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^3+3\left(\frac{1}{2}\right)^2+3\left(\frac{1}{2}\right)+1$$

$$=\frac{1}{8}+\frac{3}{4}+\frac{3}{2}+1=\frac{1+6+12+8}{8}=\frac{27}{8}$$

Thus, the required remainder = $\frac{27}{8}$

(iii) The zero of x is 0.

∴ $p(0) = (0)^3 + 3(0)^2 + 3(0) + 1$

= 0 + 0 + 0 + 1 = 1

Thus, the required remainder = 1.

(iv) The zero of x + π is -π.

$p(-\pi) = (-\pi)^3 + 3(-\pi)^2 2 + 3(-\pi) + 1$

$= -\pi^3 + 3\pi^2 + (-3\pi) + 1$

$= -\pi^3 + 3\pi^2 - 3\pi + 1$

Thus, the required remainder is $-\pi^3 + 3\pi^2 - 3\pi + 1$.

(v) The zero of 5 + 2x is $-\frac{5}{2}$.

$$\therefore \quad p\left(-\frac{5}{2}\right)=\left(-\frac{5}{2}\right)^3+3\left(-\frac{5}{2}\right)^2+3\left(\frac{-5}{2}\right)+1$$

$$=-\frac{125}{8}+3\left(\frac{25}{4}\right)+\left(-\frac{15}{2}\right)+1$$

$$=\frac{-125}{8}+\frac{75}{4}-\frac{15}{2}+1=\frac{-27}{8}$$

Thus, the required remainder is $-\frac{27}{8}$.

Question 2.

Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by x – a.

Solution:

We have, $p(x) = x^3 - ax^2 + 6x - a$ and zero of x – a is a.

∴ $p(a) = (a)^3 - a(a)^2 + 6(a) - a$

$= a^3 - a^3 + 6a - a = 5a$

Thus, the required remainder is 5a.

Question 3.

Check whether 7 + 3x is a factor of $3x^3+7x$.

Solution:

We have, $p(x) = 3x^3+7x$. and zero of 7 + 3x is $-\frac{7}{3}$.

$$\therefore \quad p\left(\frac{-7}{3}\right)=3\left(\frac{-7}{3}\right)^3+7\left(\frac{-7}{3}\right)$$
$$=3\left(\frac{-343}{27}\right)+\left(\frac{-49}{3}\right)=-\frac{343}{9}-\frac{49}{3}=-\frac{490}{9}$$

Since,(−4909) ≠ 0
i.e. the remainder is not 0.
∴ $3x^3 + 7x$ is not divisib1e by 7 + 3x.
Thus, 7 + 3x is not a factor of $3x^3 + 7x$.

NCERT So1utions for Class 9 Maths Chapter 2 Polynomials Ex 2.4

Question 1.
Determine which of the following polynomials has (x +1) a factor.
(i) $x^3+x^2+x +1$
(ii) $x^4 + x^3 + x^2 + x + 1$
(iii) $x^4 + 3x^3 + 3x^2 + x + 1$
(iv) $x^3 – x^2 – (2 +\sqrt{2})x + \sqrt{2}$
Solution:
The zero of x + 1 is -1.
(i) Let p (x) = $x^3 + x^2 + x + 1$
∴ p (-1) = $(-1)^3 + (-1)^2 + (-1) + 1$.
= -1 + 1 – 1 + 1
⇒ p (- 1) = 0
So, (x+ 1) is a factor of $x^3 + x^2 + x + 1$.

(ii) Let p (x) = $x^4 + x^3 + x^2 + x + 1$
∴ P(-1) = $(-1)^4 + (-1)^3 + (-1)^2 + (-1)+1$
= 1 – 1 + 1 – 1 + 1
⇒ P (-1) ≠ 1
So, (x + 1) is not a factor of $x^4 + x^3 + x^2 + x+ 1$.

(iii) Let p (x) = $x^4 + 3x^3 + 3x^2 + x + 1$.
∴ p (-1)= $(-1)^4 + 3 (-1)^3 + 3 (-1)^2 + (- 1) + 1$
= 1 – 3 + 3 – 1 + 1 = 1
⇒ p (-1) ≠ 0
So, (x + 1) is not a factor of $x^4 + 3x^3 + 3x^2 + x+ 1$.

(iv) Let p (x) = $x^3 – x^2 – (2 + \sqrt{2}) x + \sqrt{2}$
∴ p (- 1) =(- 1)3- (-1)2 – (2 + √2)(-1) + √2
= -1 – 1 + 2 + √2 + √2
= 2√2
⇒ p (-1) ≠ 0
So, (x + 1) is not a factor of $x^3 – x^2 – (2 + \sqrt{2}) x + \sqrt{2}$.

Question 2.
Use the Factor Theorem to determine whether g (x) is a factor of p (x) in each of the following cases
(i) p (x)= $2x^3 + x^2 – 2x – 1$, g (x) = x + 1
(ii) p(x)= $x^3 + 3x^2 + 3x + 1$, g (x) = x + 2
(iii) p (x) = $x^3 – 4x^2 + x + 6$, g (x) = x – 3
Solution:
(i) We have, p (x)= $2x^3 + x^2 – 2x – 1$ and g (x) = x + 1
∴ $p(-1) = 2(-1)^3 + (-1)^2 – 2(-1) – 1$
= 2(-1) + 1 + 2 – 1
= -2 + 1 + 2 -1 = 0
⇒ p(-1) = 0, so g(x) is a factor of p(x).

(ii) We have, p(x) $x^3 + 3x^2 + 3x + 1$ and g(x) = x + 2
∴ $p(-2) = (-2)^3 + 3(-2)^2+ 3(-2) + 1$
= -8 + 12 – 6 + 1
= -14 + 13
= -1
⇒ p(-2) ≠ 0, so g(x) is not a factor of p(x).

(iii) We have, = $x^3 – 4x^2 + x + 6$ and g (x) = x – 3
∴ $p(3) = (3)^3 – 4(3)^2 + 3 + 6$
= 27 – 4(9) + 3 + 6
= 27 – 36 + 3 + 6 = 0
⇒ p(3) = 0, so g(x) is a factor of p(x).

Question 3.
Find the value of k, if x – 1 is a factor of p (x) in each of the following cases
(i) p (x) = $x^2 + x + k$
(ii) p (x) = $2x^2 + kx + \sqrt{2}$
(iii) p (x) = $kx^2 – \sqrt{2} x + 1$
(iv) p (x) = $kx^2 – 3x + k$
Solution:
For (x – 1) to be a factor of p(x), p(1) should be equal to 0.

(i) Here, p(x) = $x^2 + x + k$
Since, $p(1) = (1)^2 +1 + k$
⇒ p(1) = k + 2 = 0
⇒ k = -2.

(ii) Here, p (x) = $2x^2 + kx + \sqrt{2}$
Since, $p(1) = 2(1)^2 + k(1) + \sqrt{2}$
= $2 + k + \sqrt{2} =0$
k = $-2 – \sqrt{2} = -(2 + \sqrt{2})$

(iii) Here, $p(x) = kx^2 - \sqrt{2}x + 1$
Since, $p(1) = k(1)^2 - (1) + 1$
$= k - \sqrt{2} + 1 = 0$
$\Rightarrow k = \sqrt{2} - 1$

(iv) Here, $p(x) = kx^2 - 3x + k$
$p(1) = k(1)^2 - 3(1) + k$
$= k - 3 + k$
$= 2k - 3 = 0$
$\Rightarrow k = 34$

Question 4.
Factorise
(i) $12x^2 - 7x + 1$
(ii) $2x^2 + 7x + 3$
(iii) $6x^2 + 5x - 6$
(iv) $3x^2 - x - 4$
Solution:
(i) We have,
$12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1$
$= 4x(3x - 1) - 1(3x - 1)$
$= (3x - 1)(4x - 1)$
Thus, $12x^2 - 7x + 3 = (2x - 1)(x + 3)$

(ii) We have, $2x^2 + 7x + 3 = 2x^2 + x + 6x + 3$
$= x(2x + 1) + 3(2x + 1)$
$= (2x + 1)(x + 3)$
Thus, $2\times2 + 7x + 3 = (2x + 1)(x + 3)$

(iii) We have, $6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$
$= 3x(2x + 3) - 2(2x + 3)$
$= (2x + 3)(3x - 2)$
Thus, $6x^2 + 5x - 6 = (2x + 3)(3x - 2)$

(iv) We have, $3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$
$= x(3x - 4) + 1(3x - 4) = (3x - 4)(x + 1)$
Thus, $3x^2 - x - 4 = (3x - 4)(x + 1)$

Question 5.
Factorise
(i) $x^3 - 2x^2 - x + 2$
(ii) $x^3 - 3x^2 - 9x - 5$
(iii) $x^3 + 13x^2 + 32x + 20$
(iv) $2y^3 + y^2 - 2y - 1$
Solution:
(i) We have, $x^3 - 2x^2 - x + 2$

Rearranging the terms, we have $x^3 – x – 2x^2 + 2$
$= x(x^2 – 1) – 2(x^2 -1) = (x^2 – 1)(x – 2)$
$= [(x)^2 – (1)^2](x – 2)$
$= (x – 1)(x + 1)(x – 2)$
$[\because (a^2 – b^2) = (a + b)(a-b)]$
Thus, $x^3 – 2x^2 – x + 2 = (x – 1)(x + 1)(x – 2)$

(ii) We have, $x^3 – 3x^2 – 9x – 5$
$= x^3 + x^2 – 4x^2 – 4x – 5x – 5$,
$= x^2 (x + 1) – 4x(x + 1) – 5(x + 1)$
$= (x + 1)(x^2 – 4x – 5)$
$= (x + 1)(x^2 – 5x + x – 5)$
$= (x + 1)[x(x – 5) + 1(x – 5)]$
$= (x + 1)(x – 5)(x + 1)$
Thus, $x^3 – 3x^2 – 9x – 5 = (x + 1)(x – 5)(x +1)$

(iii) We have, $x^3 + 13x^2 + 32x + 20$
$= x^3 + x^2 + 12x^2 + 12x + 20x + 20$
$= x^2(x + 1) + 12x(x +1) + 20(x + 1)$
$= (x + 1)(x^2 + 12x + 20)$
$= (x + 1)(x^2 + 2x + 10x + 20)$
$= (x + 1)[x(x + 2) + 10(x + 2)]$
$= (x + 1)(x + 2)(x + 10)$
Thus, $x^3 + 13x^2 + 32x + 20$
$= (x + 1)(x + 2)(x + 10)$

(iv) We have, $2y^3 + y^2 – 2y – 1$
$= 2y^3 – 2y^2 + 3y^2 – 3y + y – 1$
$= 2y^2(y – 1) + 3y(y – 1) + 1(y – 1)$
$= (y – 1)(2y^2 + 3y + 1)$
$= (y – 1)(2y^2 + 2y + y + 1)$
$= (y – 1)[2y(y + 1) + 1(y + 1)]$
$= (y – 1)(y + 1)(2y + 1)$
Thus, $2y^3 + y^2 – 2y – 1$
$= (y – 1)(y + 1)(2y +1)$

NCERT Solutions for Class 9 Maths Chapter 2 Polynomials Ex 2.5

Question 1.
Use suitable identities to find the following products
(i) $(x + 4)(x + 10)$
(ii) $(x+8) (x -10)$
(iii) $(3x + 4) (3x – 5)$
(iv) $(y^2+ 32) (y^2– 32)$

(v) (3 – 2x) (3 + 2x)
Solution:
(i) We have, (x+ 4) (x + 10)
Using identity,
$(x+ a) (x+ b) = x^2 + (a + b) x+ ab$.
We have, $(x + 4) (x + 10) = x^2+(4 + 10) x + (4 \times 10)$
$= x^2 + 14x+40$
(ii) We have, (x+ 8) (x -10)
Using identity,
$(x + a) (x + b) = x^2 + (a + b) x + ab$
We have, $(x + 8) (x – 10) = x^2 + [8 + (-10)] x + (8) (- 10)$
$= x^2 – 2x – 80$

(iii) We have, (3x + 4) (3x – 5)
Using identity,
$(x + a) (x + b) = x^2 + (a + b) x + ab$
We have, $(3x + 4) (3x – 5) = (3x)^2 + (4 – 5) x + (4) (- 5)$
$= 9x^2 – x – 20$

(iv) We have, $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

Using the identity, $(a+b)(a-b)=a^2-b^2$,
we have

$$\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)=(y^2)^2-\left(\frac{3}{2}\right)^2=y^4-\frac{9}{4}$$

(v) We have, $(3-2x)(3+2x)$
Using the identity, $(a+b)(a-b)=a^2-b^2$,
we have
$(3-2x)(3+2x)=(3)^2-(2x)^2=9-4x^2$

Question 2.
Evaluate the following products without multiplying directly
(i) 103 x 107
(ii) 95 x 96
(iii) 104 x 96
Solution:
(i)We have, 103 x 107 = (100 + 3) (100 + 7)
$= (100)^2 + (3 + 7) (100)+ (3 \times 7)$
$[\text{Using } (x + a)(x + b) = x^2 + (a + b)x + ab]$
= 10000 + (10) x 100 + 21
= 10000 + 1000 + 21=11021

(ii) We have, 95 x 96 = (100 – 5) (100 – 4)
= $(100)^2$ + [(- 5) + (- 4)] 100 + (- 5 x – 4)
[Using $(x + a)(x + b) = x^2 + (a + b)x + ab$]
= 10000 + (-9) + 20 = 9120
= 10000 + (-900) + 20 = 9120

(iii) We have 104 x 96 = (100 + 4) (100 – 4)
= $(100)^2-4^2$
[Using $(a + b)(a -b) = a^2– b^2$]
= 10000 – 16 = 9984

Question 3.
Factorise the following using appropriate identities
(i) $9x^2 + 6xy + y^2$
(ii) $4y^2-4y + 1$
(iii) x^2 – y2100
Solution:
(i) We have, $9x^2 + 6xy + y^2$
= $(3x)^2 + 2(3x)(y) + (y)^2$
= $(3x + y)^2$
[Using $a^2 + 2ab + b^2 = (a + b)^2$]
= (3x + y)(3x + y)
(ii) We have, $4y^2 – 4y + 1^2$
= $(2y)^2 + 2(2y)(1) + (1)^2$
= $(2y -1)^2$
[Using $a^2 – 2ab + b^2 = (a- b)^2$]
= (2y – 1)(2y – 1)

(iii) We have, $x^2 - \frac{y^2}{100} = (x)^2 - \left(\frac{y}{10}\right)^2$

$= \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$

[Using $a^2 - b^2 = (a+b)(a-b)$]

Question 4.
Expand each of the following, using suitable identity
(i) $(x+2y+ 4z)^2$
(ii) $(2x – y + z)^2$
(iii) $(- 2x + 3y + 2z)^2$
(iv) $(3a -7b – c)^2$
(v) $(- 2x + 5y – 3z)^2$
(vi) [14a –14b + 1] 2
Solution:
We know that
$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(i) $(x + 2y + 4z)^2$
$= x^2 + (2y)^2 + (4z)^2 + 2 (x) (2y) + 2 (2y) (4z) + 2(4z) (x)$
$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8 zx$

(ii) $(2x – y + z)^2 = (2x)^2 + (- y)^2 + z^2 + 2 (2x) (- y)+ 2 (- y) (z) + 2 (z) (2x)$
$= 4x^2 + y^2 + z^2 – 4xy – 2yz + 4zx$

(iii) $(- 2x + 3y + 2z)^2 = (- 2x)^2 + (3y)^2 + (2z)^2 + 2 (- 2x) (3y)+ 2 (3y) (2z) + 2 (2z) (- 2x)$
$= 4x^2 + 9y^2 + 4z^2 – 12xy + 12yz – 8zx$

(iv) $(3a -7b- c)^2 = (3a)^2 + (- 7b)^2 + (- c)^2 + 2 (3a) (- 7b) + 2 (- 7b) (- c) + 2 (- c) (3a)$
$= 9a^2 + 49b^2 + c^2 – 42ab + 14bc – 6ac$

(v) $(- 2x + 5y- 3z)^2 = (- 2x)^2 + (5y)^2 + (- 3z)^2 + 2 (- 2x) (5y) + 2 (5y) (- 3z) + 2 (- 3z) (- 2x)$
$= 4x^2 + 25y^2 + 9z^2 – 20xy – 30yz + 12zx$

(vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2 = \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2$
$+ 2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right) + 2\left(-\frac{1}{2}b\right)(1) + 2(1)\left(\frac{1}{4}a\right)$
$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$

Question 5.
Factorise
(i) $4 x^2 + 9y^2 + 16z^2 + 12xy – 24yz – 16xz$
(ii) $2x^2 + y^2 + 8z^2 – 2\sqrt{2}xy + 4\sqrt{2}yz – 8xz$
Solution:
(i) $4x^2 + 9y^2 + 16z^2 + 12xy – 24yz – 16xz$
$= (2x)^2 + (3y)^2 + (- 4z)^2 + 2 (2x) (3y) + 2 (3y) (- 4z) + 2 (- 4z) (2x)$
$= (2x + 3y – 4z)^2 = (2x + 3y + 4z) (2x + 3y – 4z)$

(ii) $2x^2 + y^2 + 8z^2 – 2\sqrt{2}xy + 4\sqrt{2}yz – 8xz$
$= (- \sqrt{2}x)^2 + (y)^2 + (2 \sqrt{2}z)^2y + 2(- \sqrt{2}x) (y)+ 2 (y) (2\sqrt{2}z) + 2 (2\sqrt{2}z) (- \sqrt{2}x)$
$= (- \sqrt{2}x + y + 2 \sqrt{2}z)^2$
$= (- \sqrt{2}x + y + 2 \sqrt{2}z) (- \sqrt{2}x + y + 2 \sqrt{2}z)$

Question 6.
Write the following cubes in expanded form

(i) $(2x+1)^3$ (ii) $(2a-3b)^3$

(iii) $\left(\frac{3}{2}x+1\right)^3$ (iv) $\left(x-\frac{2}{3}y\right)^3$

Solution:
We have, $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ …(1)
and $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ …(2)

(i) $(2x + 1)^3 = (2x)^3 + (1)^3 + 3(2x)(1)(2x + 1)$ [By (1)]
$= 8x^3 + 1 + 6x(2x + 1)$
$= 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a – 3b)^3 = (2a)^3 – (3b)^3 – 3(2a)(3b)(2a – 3b)$ [By (2)]
$= 8a^3 – 27b^3 – 18ab(2a – 3b)$
$= 8a^3 – 27b^3 – 36a^2b + 54ab^2$

(iii) $\left(\frac{3}{2}x+1\right)^3=\left(\frac{3}{2}x\right)^3+(1)^3+3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x+1\right)$

[By (1)]

$$=\frac{27}{8}x^3+1+\frac{9}{2}x\left[\frac{3}{2}x+1\right]$$
$$=\frac{27}{8}x^3+1+\frac{27}{4}x^2+\frac{9}{2}x$$
$$=\frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$$

(iv) $\left(x-\frac{2}{3}y\right)^3=x^3-\left(\frac{2}{3}y\right)^3-3(x)\left(\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$

[By (2)]

$$= x^3-\frac{8}{27}y^3-2xy\left(x-\frac{2}{3}y\right)$$
$$=x^3-\frac{8}{27}y^3-2x^2y+\frac{4}{3}xy^2$$

Question 7.
Evaluate the following using suitable identities
(i) $(99)^3$
(ii) $(102)^3$
(iii) $(998)^3$
Solution:
(i) We have, 99 = (100 -1)

∴ $99^3 = (100 – 1)^3$
$= (100)^3 – 1^3 – 3(100)(1)(100 -1)$
[Using $(a – b)^3 = a^3 – b^3 – 3ab (a – b)$]
= 1000000 – 1 – 300(100 – 1)
= 1000000 -1 – 30000 + 300
= 1000300 – 30001 = 970299

(ii) We have, 102 =100 + 2
∴ $102^3 = (100 + 2)^3$
$= (100)^3 + (2)^3 + 3(100)(2)(100 + 2)$
[Using $(a + b)^3 = a^3 + b^3 + 3ab (a + b)$]
= 1000000 + 8 + 600(100 + 2)
= 1000000 + 8 + 60000 + 1200 = 1061208

(iii) We have, 998 = 1000 – 2
∴ $(998)^3 = (1000-2)^3$
$= (1000)^3– (2)^3 – 3(1000)(2)(1000 – 2)$
[Using $(a – b)^3 = a^3 – b^3 – 3ab (a – b)$]
= 1000000000 – 8 – 6000(1000 – 2)
= 1000000000 – 8 – 6000000 +12000
= 994011992

Question 8.
Factorise each of the following
(i) $8a^3 +b^3 + 12a^2b+6ab^2$
(ii) $8a^3 -b^3-12a^2b+6ab^2$
(iii) $27-125a^3 -135a+225a^2$
(iv) $64a^3 -27b^3 -144a^2b + 108ab^2$
(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$
Solution:
(i) $8a^3 +b^3 +12a^2b+6ab^2$
$= (2a)^3 + (b)^3 + 6ab(2a + b)$
$= (2a)^3 + (b)^3 + 3(2a)(b)(2a + b)$
$= (2 a + b)^3$
[Using $a^3 + b^3 + 3 ab(a + b) = (a + b)^3$]
= (2a + b)(2a + b)(2a + b)

(ii) $8a^3 – b^3 – 12o^2b + 6ab^2$
$= (2a)^3 – (b)^3 – 3(2a)(b)(2a – b)$
$= (2a – b)^3$
[Using $a^3 + b^3 + 3 ab(a + b) = (a + b)^3$]
= (2a – b) (2a – b) (2a – b)

(iii) $27 - 125a^3 - 135a + 225a^2$
$= (3)^3 - (5a)^3 - 3(3)(5a)(3 - 5a)$
$= (3 - 5a)^3$
[Using $a^3 + b^3 + 3ab(a + b) = (a + b)^3$]
$= (3 - 5a)(3 - 5a)(3 - 5a)$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$
$= (4a)^3 - (3b)^3 - 3(4a)(3b)(4a - 3b)$
$= (4a - 3b)^3$
[Using $a^3 - b^3 - 3ab(a - b) = (a - b)^3$]
$= (4a - 3b)(4a - 3b)(4a - 3b)$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$
$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)\left(\frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$
$= \left(3p - \frac{1}{6}\right)^3$
[Using $a^3 - b^3 - 3ab(a - b) = (a - b)^3$]
$= \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

Question 9.
Verify
(i) $x^3 + y^3 = (x + y)\text{-}(x^2 - xy + y^2)$
(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
Solution:
(i) $\because (x + y)^3 = x^3 + y^3 + 3xy(x + y)$
$\Rightarrow (x + y)^3 - 3(x + y)(xy) = x^3 + y^3$
$\Rightarrow (x + y)[(x + y)2-3xy] = x^3 + y^3$
$\Rightarrow (x + y)(x^2 + y^2 - xy) = x^3 + y^3$
Hence, verified.

(ii) $\because (x - y)^3 = x^3 - y^3 - 3xy(x - y)$
$\Rightarrow (x - y)^3 + 3xy(x - y) = x^3 - y^3$
$\Rightarrow (x - y)[(x - y)^2 + 3xy)] = x^3 - y^3$
$\Rightarrow (x - y)(x^2 + y^2 + xy) = x^3 - y^3$
Hence, verified.

Question 10.
Factorise each of the following
(i) $27y^3 + 125z^3$

(ii) $64m^3 – 343n^3$
[Hint See question 9]
Solution:
(i) We know that
$x^3 + y^3 = (x + y)(x^2 – xy + y^2)$
We have, $27y^3 + 125z^3 = (3y)^3 + (5z)^3$
$= (3y + 5z)[(3y)^2 – (3y)(5z) + (5z)^2]$
$= (3y + 5z)(9y^2 – 15yz + 25z^2)$

(ii) We know that
$x^3 – y^3 = (x – y)(x^2 + xy + y^2)$
We have, $64m^3 – 343n^3 = (4m)^3 – (7n)^3$
$= (4m – 7n)[(4m)^2 + (4m)(7n) + (7n)^2]$
$= (4m – 7n)(16m^2 + 28mn + 49n^2)$

Question 11.
Factorise $27x^3 +y^3 +z^3 -9xyz$.
Solution:
We have,
$27x^3 + y^3 + z^3 – 9xyz = (3x)^3 + (y)^3 + (z)^3 – 3(3x)(y)(z)$
Using the identity,
$x^3 + y^3 + z^3 – 3xyz = (x + y + z)(x^2 + y^2 + z^2 – xy – yz – zx)$
We have, $(3x)^3 + (y)^3 + (z)^3 – 3(3x)(y)(z)$
$= (3x + y + z)[(3x)^3 + y^3 + z^3 – (3x × y) – (y × 2) – (z × 3x)]$
$= (3x + y + z)(9x^2 + y^2 + z^2 – 3xy – yz – 3zx)$

Question 12.
Verify that
$x^3 +y^3 +z^3 – 3xyz = 12 (x + y+z)[(x-y)^2 + (y – z)^2 +(z – x)^2]$
Solution:
R.H.S
$= 12(x + y + z)[(x – y)^2+(y – z)^2+(z – x)^2]$
$= 12 (x + y + 2)[(x^2 + y^2 – 2xy) + (y^2 + z^2 – 2yz) + (z^2 + x^2 – 2zx)]$
$= 12 (x + y + 2)(x^2 + y^2 + y^2 + z^2 + z^2 + x^2 – 2xy – 2yz – 2zx)$
$= 12 (x + y + z)[2(x^2 + y^2 + z^2 – xy – yz – zx)]$
$= 2$ x 12 x $(x + y + z)(x^2 + y^2 + z^2 – xy – yz – zx)$
$= (x + y + z)(x^2 + y^2 + z^2 – xy – yz – zx)$
$= x^3 + y^3 + z^3 – 3xyz$ = L.H.S.
Hence, verified.
Question 13.
If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3 xyz$.
Solution:
Since, $x + y + z = 0$
$\Rightarrow x + y = -z \; (x + y)^3 = (-z)^3$
$\Rightarrow x^3 + y^3 + 3xy(x + y) = -z^3$
$\Rightarrow x^3 + y^3 + 3xy(-z) = -z^3 \; [\because x + y = -z]$

⇒ $x^3 + y^3 - 3xyz = -z^3$
⇒ $x^3 + y^3 + z^3 = 3xyz$
Hence, if $x + y + z = 0$, then
$x^3 + y^3 + z^3 = 3xyz$

Question 14.
Without actually calculating the cubes, find the value of each of the following
(i) $(-12)^3 + (7)^3 + (5)^3$
(ii) $(28)^3 + (-15)^3 + (-13)^3$
Solution:
(i) We have, $(-12)^3 + (7)^3 + (5)^3$
Let $x = -12$, $y = 7$ and $z = 5$.
Then, $x + y + z = -12 + 7 + 5 = 0$
We know that if $x + y + z = 0$, then, $x^3 + y^3 + z^3 = 3xyz$
∴ $(-12)^3 + (7)^3 + (5)^3 = 3[(-12)(7)(5)]$
$= 3[-420] = -1260$

(ii) We have, $(28)^3 + (-15)^3 + (-13)^3$
Let $x = 28$, $y = -15$ and $z = -13$.
Then, $x + y + z = 28 - 15 - 13 = 0$
We know that if $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$
∴ $(28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$
$= 3(5460) = 16380$

Question 15.
Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given
(i) Area $25a^2 - 35a + 12$
(ii) Area $35y^2 + 13y - 12$
Solution:
Area of a rectangle = (Length) x (Breadth)
(i) $25a^2 - 35a + 12 = 25a^2 - 20a - 15a + 12 = 5a(5a - 4) - 3(5a - 4) = (5a - 4)(5a - 3)$
Thus, the possible length and breadth are $(5a - 3)$ and $(5a - 4)$.

(ii) $35y^2 + 13y - 12 = 35y^2 + 28y - 15y - 12$
$= 7y(5y + 4) - 3(5y + 4) = (5y + 4)(7y - 3)$
Thus, the possible length and breadth are $(7y - 3)$ and $(5y + 4)$.

Question 16.
What are the possible expressions for the dimensions of the cuboids whose volumes are given below?
(i) Volume $3x^2 - 12x$
(ii) Volume $12ky^2 + 8ky - 20k$
Solution:
Volume of a cuboid = (Length) x (Breadth) x (Height)

(i) We have, $3x^2 - 12x = 3(x^2 - 4x)$
$= 3 \times x \times (x - 4)$
∴ The possible dimensions of the cuboid are 3, x and (x – 4).

(ii) We have, $12ky^2 + 8ky - 20k$
$= 4[3ky^2 + 2ky - 5k] = 4[k(3y^2 + 2y - 5)]$
$= 4 \times k \times (3y^2 + 2y - 5)$
$= 4k[3y^2 - 3y + 5y - 5]$
$= 4k[3y(y - 1) + 5(y - 1)]$
$= 4k[(3y + 5) \times (y - 1)]$
$= 4k \times (3y + 5) \times (y - 1)$
Thus, the possible dimensions of the cuboid are 4k, (3y + 5) and (y -1).